Roll No. .....

E–523

M. A./M. Sc. (Second Semester) (Main/ATKT)

EXAMINATION, May-June, 2021

MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics—II)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. A vertex of degree zero is called :
   (a) Pendant vertex
   (b) Even vertex
   (c) Isolated vertex
   (d) None of the above

2. A graph G is said to be ..... if every vertex in G is connected to every other vertex in G.
   (a) Complete graph
   (b) Regular graph
   (c) Bipartite graph
   (d) Isomorphic graph

3. The maximum number of edges in an *n*-node undirected graph without self-loop is :
   (a) $n-1$
   (b) $\frac{n(n-1)}{2}$
   (c) $\frac{n(n+1)}{2}$
   (d) $n+1$

4. A connected planar graph having 12 edges, 4 regions, contains vertices :
   (a) 15
   (b) 12
   (c) 16
   (d) 10

5. A connected graph is said to be tree if it has :

   (a) One circuit

   (b) Two circuits

   (c) Many circuits

   (d) No circuit

6. A cutset of graph is a set of :

   (a) Edges

   (b) Vertices

   (c) Edges and vertices

   (d) None of the above

7. Which of the given statements is correct ?

   (a) A graph is bipartite iff all its circuits are of odd length.

   (b) In any graph the number of vertices of odd degree is always odd.

   (c) A tree with n vertices has $n + 1$ edges.

   (d) A given connected graph G is a Euler graph iff all vertices of G are of even degree.

8. In a directed graph with e edges, which of the given are correct ?

   (a) $\sum_{i=1}^{n} d^{-}(v_i) < e$ and $\sum_{i=1}^{n} d^{+}(v_i) < e$

   (b) $\sum_{i=1}^{n} d^{-}(v_i) = e$ and $\sum_{i=1}^{n} d^{+}(v_i) = e$

   (c) $\sum_{i=1}^{n} d^{-}(v_i) > e$ and $\sum_{i=1}^{n} d^{+}(v_i) > e$

   (d) $\sum_{i=1}^{n} d^{-}(v_i) < e$ and $\sum_{i=1}^{n} d^{+}(v_i) > e$

9. A vertex v of a graph with zero out degree is called :

   (a) Trial

   (b) Source

   (c) Sink

   (d) None of the above

10. In a binary tree if there are l leaves, then total number of nodes n is :

    (a) $n = l - 1$

    (b) $n = 2l - 1$

    (c) $n = 2l$

    (d) $n = l + 1$

11. For finding the minimum spanning tree we use :

    (a) Dijkstra's algorithm

    (b) Warshall's algorithm

    (c) Kruskal's algorithm

    (d) Graph algorithm

12. If G is a finite directed graph, then it is strongly connected iff :

    (a) G has closed spanning path

    (b) G has open spanning path

    (c) G has a spanning path

    (d) G has a spanning semipath

13. The relation K-equivalent on the set S of all states of M is :

    (a) Only reflexive relation

    (b) An equivalence relation

    (c) Only symmetric relation

    (d) Only transitive relation

14. The finite automata is called NFA when there exists ..... for a specific input from current state to next state.

    (a) Multiple paths

    (b) Only one path

    (c) Two paths

    (d) None of these

15. A mealy machine M consists of :

    (a) Five parts

    (b) Four parts

    (c) Seven parts

    (d) Six parts

16. Which of the following options is correct ?

    Statement A : The final state of DFA will be every combination of final state of NFA.

    Statement B : Initial state of NFA is initial state of DFA.

    (a) Statement (A) is true but (B) is false.

    (b) Statement (A) is false but (B) is true.

    (c) Both statements (A) and (B) are false.

    (d) Both statements (A) and (B) are true.

17. A finite state machine in which :

    (i) the output is a function of only the current state,

    (ii) the output is a function of the current state and inputs.

    Which of the following machine is respectively correct for these styles ?

    (a) Mealy machine and Moore machine

    (b) State machine and Moore machine

    (c) Moore machine and Mealy machine

    (d) Mealy machine and State machine

18. NFA in its name has non-deterministic because of :

    (a) The state to be transited next is non-deterministic

    (b) The choice of path is non-deterministic

    (c) The result is undetermined

    (d) All of the mentioned

19. Moore machine is an application of :

    (a) Finite automata with output

    (b) Non-finite automata with output

    (c) Finite automata without input

    (d) Non-finite automata with input

20. A machine M involves 3 disjoint non-empty sets : (i) A finite tape set, (ii) A finite state set and (iii) A direction set. This type of machine is called :

    (a) Mealy machine

    (b) Moore machine

    (c) Turing machine

    (d) State machine

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions in 2-3 sentences.

1. Define complete biapartite graph.
2. Write the statement of Kuratowski theorem.

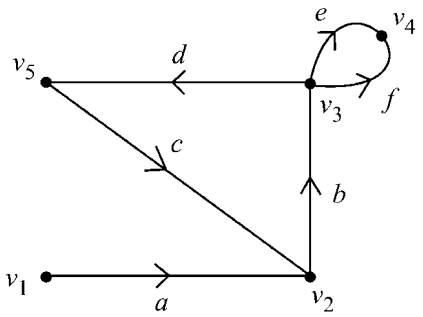
3. Define the cutsets of graph.
4. Define weighted undirected graph.
5. Write the statement of Kleene's theorem.
6. Define Binary tree.
7. Define equivalent machines.
8. Define Mealy machine.

Section—C 3 each

(Short Answer Type Questions)

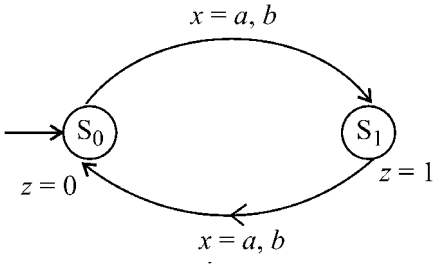
Note : Attempt all questions in less than 75 words.

1. Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.
2. Write the incidence matrix of the graph :



3. Explain tree traversals.
4. Define spanning tree with an example.
5. Define finite state machine with example.

6. Define non-deterministic Finite Automata with example.
7. Explain the Dijkstra's algorithm and give an example.
8. Construct the transition and output table for Moore machine given in the figure :

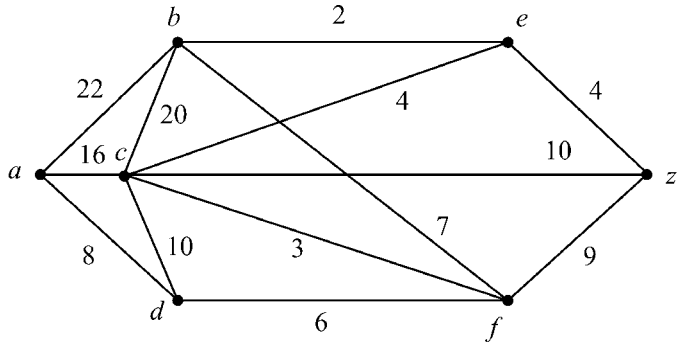


Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Write an algorithm for shortest path in weighted graph and use it to find shortest path from *a* to *z* in the graph shown in figure where the number associated with edges are the weights :



Or

Define and give example of each of the following :

(a) Isomorphic graph

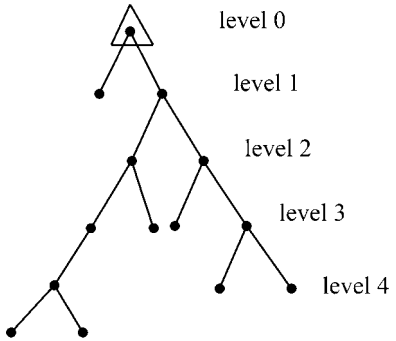
(b) Planar graph

(c) Hamiltonian circuit

(d) Degree of graph

(e) Pendant vertex

2. Find the path length of the binary tree in the following figure :



Or

Design a Mealy machine that prints 1's complement of an input its string.

3. Draw the graph having the following matrix as its adjacency matrix :

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 3 \\ 2 & 0 & 2 & 3 \\ 0 & 2 & 0 & 2 \\ 1 & 3 & 2 & 2 \end{bmatrix} \end{array}$$

Or

Prove that a connected planar graph with n vertices and e edges has r regions given by $r = e - n + 2$.

4. Minimize finite state machine M, where M is given by the following state table :

State	Input		Output
S_0	S_3	S_1	1
S_1	S_4	S_1	0
S_2	S_3	S_0	1
S_3	S_2	S_3	0
S_4	S_1	S_0	1

Or

Consider the Moore machine described by the transition table given below. Construct the corresponding Mealy machine :

Present State	Next State		Output
	$a = 0$	$a = 1$	
S_1	S_1	S_2	0
S_2	S_1	S_3	0
S_3	S_1	S_3	1