Roll No. .....

E–1000

M. A./M. Sc. (Fourth Semester) (Main/ATKT)

EXAMINATION, May-June, 2021

MATHEMATICS

Paper Fifth

(Optional—B)

(Graph Theory—II)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. A graph G is called split graph, if :
   (a) $V = S \cup K$
   (b) $V = S \cap K$
   (c) Both (a) and (b)
   (d) None of the above

2. The sum of the in-degree and out-degree of a vertex is called the :
   (a) source
   (b) sink
   (c) total degree
   (d) None of the above
3. A graph which has neither loops nor multiple edges is called :
   (a) Multigraph
   (b) Simple graph
   (c) Pseudo graph
   (d) None of the above
4. What is the definition of graph according to graph theory ?
   (a) Visual representation of data
   (b) Collection of dots and lines
   (c) Collection of edges
   (d) Collection of vertices
5. What will be the chromatic number for a line graph having *n*-vertices ?
   (a) 0
   (b) 1
   (c) 2
   (d) *n*

6. A non-planar graph can have :
   (a) Complete graph
   (b) Line graph
   (c) Sub graph
   (d) Bar graph

7. The number of different rooted, labelled tree with n-vertices is :
   (a) n^{n-2}
   (b) n^{n-1}
   (c) n^n
   (d) n

8. The number of colours used by a proper coloring graph is called :
   (a) k-coloring graph
   (b) X-coloring graph
   (c) m-coloring graph
   (d) n-coloring graph

9. The graph representing universal relation is called :
   (a) Complete digraph
   (b) Partial digraph
   (c) Empty digraph
   (d) Partial subgraph

10. A network has 8 nodes and 5 independent loops. The number of branches in the network is :
    (a) 11
    (b) 12
    (c) 8
    (d) 6

11. Every weak isograph is :
    (a) Eulerian
    (b) Weak
    (c) Strong
    (d) None of the above

12. A graph which contains isolated node is called :
    (a) Regular graph
    (b) Complete graph
    (c) Simple graph
    (d) Null graph

13. A digraph is unilateral iff its transitive closure is :
    (a) Complete symmetric
    (b) Unique completion
    (c) Both (a) and (b)
    (d) Complete

14. A digraph without cycles is called :

    (a) tree

    (b) forest

    (c) acyclic

    (d) None of the above

15. Every acyclic digraph has unique :

    (a) No-basis

    (b) 1-basis

    (c) 2-basis

    (d) 3-basis

16. Every vertex of a composite connected graph lies on a :

    (a) 4-cycle

    (b) 3-cycle

    (c) 2-cycle

    (d) 1-cycle

17. Which of the following has maximum clicue size 2 ?

    (a) Perfect graph

    (b) Tree

    (c) Histogram

    (d) Cartesian

18. The number of simple digraphs with $|V| = 3$ and exactly 3 edges is :

    (a) 80

    (b) 84

    (c) 88

    (d) 92

19. Every interval graph is :

    (a) Perfect

    (b) Triangulated

    (c) Both (a) and (b)

    (d) None of the above

20. For a digraph D which is true ?

    (a) D is acyclic.

    (b) D* is isomorphic to D.

    (c) Every walk of D is a path.

    (d) All of the above

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Describe Ramsey number.
2. Describe automorphism group.
3. Describe the color polynomials.

4. Describe capacited network.
5. Describe flows in network.
6. Describe digraph.
7. Describe bipartite graph.
8. Describe the chromatically unique graph.

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt any *eight* questions.

1. Describe forbidden subgraph orientations.
2. Describe spectral studies of the automorphism group.
3. Describe graph enumeration.
4. Describe triangulated graph.
5. Describe covers and basis.
6. Describe acyclic digraph.
7. Describe perfect graphs.
8. Describe graph enumeration.
9. Describe monochromatic subset.

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. For any $S \geq 2$, prove that $R(S,S) \geq 2^{S/2}$.

Or

Prove that :

$$R_k(3) \, L k \, ! e_J + 1$$

2. Prove that every group is isomorphic to the automorphism group of some graph.

Or

If the eigen value of a graph of all distinct, then $\Gamma(G)$ is abelian and every element of Γ has order 2.

3. Prove that for any graph G the chromatic polynomial :

$$\phi(G, x) = (-1)^V x^k \, T(G, 1 - x, 0)$$

Or

Prove that the Tutte polynomial is the same as the dichromatic.

4. Prove that a non-trivial weak digraph is an isograph iff it is the union of arc-disjoint cycles.

Or

Prove that the transportation network has a feasible flow iff $d(Y \cap \bar{S}) - S(X \cap \bar{S}) \leq C(SS)$ for every subset S of V.