Roll No. .....

E–522

M. A./M. Sc. (Second Semester) (Main/ATKT) EXAMINATION, May-June, 2021

MATHEMATICS

Paper Fourth

[Advanced Complex Analysis (II)]

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer out of four alternative answers :

1. Which of the following is entire function ?

   (a) Transcendental function

   (b) Polynomial

   (c) Algebraic function

   (d) None of the above

2. $\Gamma(z) = \lim_{n\to\infty} \frac{n!n^z}{z(z+1).....(z+n)}$ is known as :

   (a) Gauss's formula

   (b) Functional equation

   (c) Legendre's duplication formula

   (d) None of the above

3. Value of $\zeta(z)$ is :

   (a) $\zeta(z) = \sum_{n=1}^{\infty} z^n$

   (b) $\zeta(z) = \sum_{n=1}^{\infty} n^z$

   (c) $\zeta(z) = \sum_{n=1}^{\infty} n^{-z}$

   (d) None of the above

4. The value of $E_p(1) =$:

   (a) 1

   (b) 3

   (c) 0

   (d) 2

5. An analytic function *f* with its domains i.e. (*f*, D) is called :

   (a) Analytic continuation

   (b) Function element

   (c) Direct analytic continuation

   (d) None of the above

6. The collection of all function elements is called :

   (a) Germ

   (b) Genus

   (c) Univalent function

   (d) None of the above

7. Germ of f at a i.e. $[f]_a$ is :

   (a) a function element

   (b) not a function element

   (c) Both (a) and (b)

   (d) None of the above

8. Analytic continuation of an analytic function is :

   (a) unique

   (b) non unique

   (c) vanish

   (d) None of the above

9. If $f : G \to \mathbf{C}$ is an analytic function, then u = Re f and v = Im f are called :

   (a) Harmonic function

   (b) Laplace equation

   (c) Poisson kernel

   (d) Harmonic conjugates

10. If Pr is a periodic in θ with period 2π and $\Pr(\theta) > 0$, for all θ, then :

    (a) $\Pr(\theta) = -\Pr(\theta)$

    (b) $\Pr(-\theta) = \Pr(\theta)$

    (c) $\Pr(-\theta) = -\Pr(\theta)$

    (d) $-\Pr(\theta) = \Pr(2\pi - \theta)$

11. Perron family consists of :

    (a) All subharmonic functions

    (b) All superhharmonic functions

    (c) Harmonic functions

    (d) None of the above

12. If $g_a : G \to R$ is a Green's function with singularity at $a \in G$, then for each $w \in \partial_\infty G$, $\lim_{z \to w} g_a(z) =$:

    (a) 1

    (b) –1

    (c) 0

    (d) 2

13. Order of e^{z^λ} (λ is +ve integer) is :

    (a) 1

    (b) 0

    (c) λ

    (d) 2

14. The genus and the order of an entire function satisfy the inequality :

(a) $h \leq \lambda \leq h + 1$

(b) $h \geq \lambda \geq h + 1$

(c) $h + 1 \leq \lambda \leq h$

(d) None of the above

15. The sufficient condition for a real function $f(x)$ to be convex is :

(a) $f''(x) \leq 0$

(b) $f''(x) \geq 0$

(c) $f''(x) = 0$

(d) $f''(x)$ does not exist

16. If p is the rank of f and q is the degree of polynomial g, then $u = \max(p, q)$ is called :

(a) germ of g

(b) genus of f

(c) growth of f

(d) None of the above

17. Condition for univalent function is :

(a) $z_1 \neq z_2 \Rightarrow f(z_1) \neq f(z_2)$

(b) $z_1 = z_2 \Rightarrow f(z_1) \neq f(z_2)$

(c) $z_1 \neq z_2 \Rightarrow f(z_1) = f(z_2)$

(d) None of the above

18. If f is an entire function that omits two values, then :

(a) f is not constant

(b) f is constant

(c) f is non-linear

(d) None of the above

19. A univalent function that maps $|z| < \infty$ onto $|w| < \infty$ must be :

(a) zero

(b) equal

(c) linear

(d) non-linear

20. "If f has an isolated singularity at $z = z_0$ and if there are two complex numbers that are not assumed infinitely often by f, then $z = z_0$ is either a pole or removable singularity." Is statement of :

(a) Little Picard theorem

(b) Great Picard theorem

(c) Bloch's theorem

(d) $\frac{1}{4}$-theorem

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions in 2-3 sentences.

1. Define Weirstrass primary factor.
2. Define analytic continuation along path.

3. Write the statement of Monodromy theorem.
4. Define canonical product.
5. Explain exponent of convergence.
6. Define Dirichlet region.
7. State Schwarz reflection principle.
8. State the Bieberbach conjecture.

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. If $|z| \leq 1$ and $p \geq 0$, then $|1 - E_p(z)| \leq |z|^{p+1}$.
2. Define Euler's Gamma function and write any *two* properties of Gamma function.
3. Let $u : G \to R$ be a harmonic function and let $\overline{B}(a, r)$ be a closed disc such that $\overline{B}(a; r) \subset G$. If γ is the circle $|z - a| = r$, then :

$$u(a) = \frac{1}{2\pi}\int_0^{2\pi} u(a + re^{i\theta})\, d\theta$$

4. State and prove Jensen's inequality.
5. Let G be a bounded Dirichlet region. Then for each $a \in G$, there is a Green's function on G with singularity at *a*.
6. If $f(z)$ is an entire function of order ρ and convergence exponent σ, then $\sigma \leq \rho$.

7. If ρ be the order of an integral function $f(z)$, then :

$$\rho = \limsup_{r \to \infty} \frac{\log \log M(r)}{\log r}$$

where $M(r) = \max |f(z)|$ or $|z| = r$.

8. Let g be analytic in B (0; R), $g(0) = 0$, $|g'(0)| = \mu > 0$, $|g(z)| \leq M$, for all *z*. Then :

$$g(B(O; R)) \supset B\left(O; \frac{R^2\mu^2}{6M}\right)$$

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. State and prove Legendre's duplication formula.

Or

State and prove Runge's theorem.

2. State and prove Mittag-Leffler's theorem.

Or

State and prove Harnack's inequality.

3. State and prove Poisson-Jensen formula.

Or

State and prove Borel's theorem.

4. State and prove Bolch's theorem.

Or

State and prove $\frac{1}{4}$-theorem.